REFERENCE FRAME

What Is Quantum Theory?

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Over the period 1885–1889, Heinrich Hertz[1] discovered that electromagnetic waves propagate through empty space, and demonstrated experimentally that these waves travel at the speed of light and are transversely polarized. This work confirmed predictions James Clerk Maxwell had made 20 years earlier, in 1864. Hertz's major papers were collected in a book, *Electric Waves*, for which he wrote an extensive introduction. In that introduction occurs his famous, extraordinary statement: "To the question: 'What is Maxwell's theory?' I know of no shorter or more definite answer than the following: 'Maxwell's theory is Maxwell's system of equations.' "

Superficially, this statement might appear innocent—even ingenuous—but it goes deep, and in its time it caused a sensation. There was, at the time, a rival tradition of electromagnetic theories, especially strong in Germany, which advocated action-at-a-distance formulations in preference to fields. These theories had the advantage of continuing the tremendously successful Newtonian tradition, and of using familiar, highly developed mathematical methods. They also had enormous flexibility. With velocity-dependent force laws, most of the previously known facts about electricity and magnetism could readily be described using action-at-a-distance. Arnold Sommerfeld recounts[2] of his student days (1887–1889) in Koenigsberg, "The total picture of electrodynamics thus presented to us was awkward, incoherent, and by no means self-contained."

Perhaps some modification would also describe Hertz's new results. (Indeed, we know now that by using retarded potentials one *can* reproduce the Maxwell equations from an action-at-a-distance theory, rather elegantly in fact.) So Hertz sought to forestall unproductive debates between rival theories with identical physical content by focusing on the bottom-line content. Sommerfeld continues, "When I read Hertz's great paper, it was as though scales fell from my eyes."

Also, Hertz wanted to purify Maxwell's work itself. The point is that Maxwell reached his equations through a complex process of constructing and modifying mechanical models of the ether and, according to Hertz, ". . . when Maxwell composed his great treatise, the accumulated hypotheses of his earlier mode of conception no longer suited him, or else he discovered contradictions in them and so abandoned them. But he did not eliminate them completely . . ."

Yet a modern physicist, while not contradicting it, could not rest entirely satisfied with Hertz's answer to his question. Maxwell's theory is much more than Maxwell's equations. Or, to put it differently, merely writing down Maxwell's equations, and doing them justice, are two quite different things.

Indeed, a modern physicist, asked what is Maxwell's theory, might be more inclined to answer that it is special relativity plus gauge invariance. While not altering Maxwell's equations, in a real sense these concepts tell us *why* that superficially complicated system of partial differential equations must take precisely the form it does, what its essential nature is, and how it might be generalized. This last feature bears abundant fruit in the modern Standard Model. The core of the Standard Model is a mighty generalization of gauge invariance, which provides successful descriptions of physical phenomena far beyond anything Maxwell or Hertz could have imagined.

With this history as background, let us return to the analogous question, posed in my title, What is Quantum Theory? At one level, we can answer along the lines of Hertz. Quantum theory is the theory you find written down in textbooks of quantum theory. Perhaps its definitive exposition is Dirac's book.[3] Conversely, you can find, in the early parts of Dirac's book, statements very much in the Hertzian spirit:

> "The new scheme becomes a precise physical theory when all the axioms and rules of manipulation governing the mathematical quantities are specified and when in addition certain laws are laid down connecting physical facts with the mathematical formalism, so that from any given physical conditions equations between the mathematical quantities may be inferred and vice versa."

Of course, the equations of quantum theory are notoriously less straightforward to interpret than Maxwell's equations. The leading interpretations of quantum theory introduce concepts that are extrinsic to its equations ("observers"), or even contradict them ("collapse of the wave function"). The relevant literature is famously contentious and obscure. I believe it will remain so until someone constructs, within the formalism of quantum mechanics, an "observer," that is, a model entity whose states correspond to a recognizable caricature of conscious awareness; and demonstrates that the perceived interaction of this entity with the physical world, following the equations of quantum theory, accords with our experience. That is a formidable project, extending well beyond what is conventionally considered physics. Like most working physicists, I assume, perhaps naively, that this project can be accomplished, and that the equations will survive its completion unscathed. In any case, only after its successful completion might one legitimately claim that quantum theory is defined by the equations of quantum theory.

Stepping now toward firmer ground, let us consider the equations themselves. The pith of quantum theory, which plays for it the central role analogous to the role of Maxwell's

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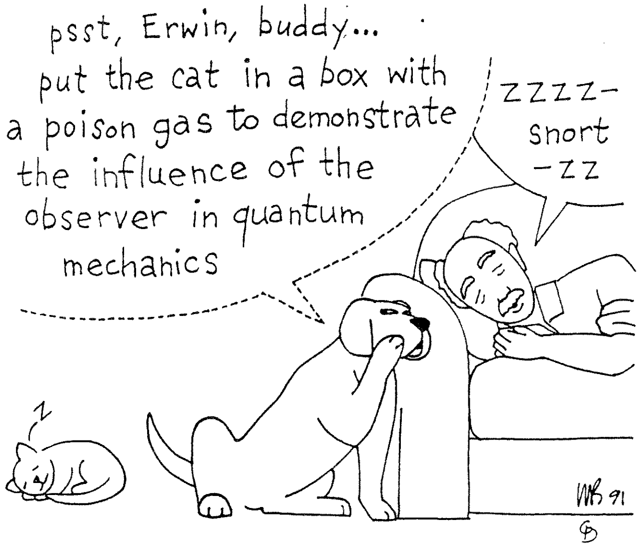

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equations in electrodynamics, is supplied by the commutation relations among dynamical variables. Specifically, it is in these commutation relations—and, ultimately, only here—that Planck's constant appears. The most familiar commutation relation, $[p, q] = -i\hbar$, is between linear momentum and position, but there are also different ones between spins, or between fermion fields. In formulating these commutation relations, the founders of quantum theory were guided by analogy, by aesthetics, and—ultimately—by a complex dialogue with Nature, through experiment. Here is how Dirac describes the crucial step:[4]

> "The problem of finding quantum conditions is not of such a general character . . . . It is instead a special problem which presents itself with each particular dynamical system one is called upon to study. . . . a fairly general method of obtaining quantum conditions . . . is the method of *classical analogy* [original italics]."

I think it is fair to say that one does not find here a profound guiding principle, comparable to the equivalence of different observers (that inspires both relativity theories) or of different potentials (that implement gauge invariance).

Those profound guiding principles of physics are statements of *symmetry*. Is it possible to phrase the equations of quantum theory as statements of symmetry? A very interesting but brief and inconclusive discussion of this occurs in Herman Weyl's singular text, where he proposes [the original is entirely italics!]:[5] "The kinematical structure of a physical system is expressed by an irreducible unitary projective representation of abelian rotations in Hilbert space."

Naturally, I won't be able to unpack this formulation here, but three comments do seem appropriate. First, Weyl shows that his formulation contains the Heisenberg algebra of quantum mechanics and the quantization of boson and fermion fields as special cases, but also allows additional possibilities. Second, the sort of symmetry he proposes—abelian—is the simplest possible kind. Third, his symmetry of quantum kinematics is entirely separate and independent from the other symmetries of physics.

The next level in understanding may come when an overarching symmetry is found, melding the conventional symmetries and Weyl's symmetry of quantum kinematics (made more specific, and possibly modified) into an organic whole. Perhaps Weyl himself anticipated this possibility, when he signed off his pioneering discussion with: "It seems more probable that the scheme of quantum kinematics will share the fate of the general scheme of quantum mechanics: to be submerged in the concrete physical laws of the only existing physical structure, the actual world."

To summarize, I feel that after seventy-five years—and innumerable successful applications—we are still two big steps away from understanding quantum theory properly.

References

1. An excellent recent brief biography of Hertz, including an extensive selection of his original papers and contemporary commentary, is J. Mulligan, *Heinrich Rudolf Hertz*, Garland (New York), 1994.
2. A. Sommerfeld, *Electrodynamics*, Academic (London), 1964, p. 2.
3. P. A. M. Dirac, *Quantum Mechanics*, 4th revised edition, Oxford (London), 1967, p. 15.
4. Ibid., p. 84.
5. H. Weyl, *The Theory of Groups and Quantum Mechanics*, Dover (New York), 1950, p. 272. ■